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# Sensor fault-tolerant observer applied in satellite attitude control

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**Abstract:** The observing failure and feedback instability might happen when the partial sensors of a satellite attitude control system (SACS) go wrong. A fault diagnosis and isolation (FDI) method based on a fault observer is introduced to detect and isolate the fault sensor at first. Based on the FDI result, the object system state-space equation is transformed and divided into a corresponsive triangular canonical form to decouple the normal subsystem from the fault subsystem. And then the *KX* fault-tolerant observers of system in different modes are designed and embedded into online monitoring. The outputs of all *KX* fault-tolerant observers are selected by the control switch process. That can make sense that the SACS is part-observed and in stable when the partial sensors break down. Simulation results demonstrate the effectiveness and superiority of the proposed method.

**Keywords:** triangular canonical form, *KX* observer, satellite attitude control, integrity in close-loop.

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## 1. Introduction

As a classical dynamical control system, the satellite attitude control system (SACS) is an important subsystem which makes the satellite running normally. For the particularities of aerospace engineering, the SACS requires extremely high reliability because any fault may affect the security of the system. Accordingly, it is indispensable to observe the state of SACS in real-time and ensure that the system can still run when faults occur [1]. For a typical control system, the model observer method is adopted to fulfill state-based feedback control and fault diagnosis (FD) with the residual error. The inputs of observers are control inputs and measured outputs of object systems and the output is the estimated state of object systems [2]. For the SACS, less redundant sensors are available due to the restrictions in aero space while the analytical redundancy exists among measured outputs; for actuator faults, the analytical redundancy can be used to implement the fault-tolerant. But for sensor faults, there exists a potential fault couple problem. Partial sensor faults will result in invalid observation when traditional model observer methods are applied in the attitude state observing. Consequently, it will lead to faulty state tracking and then unstable closed-loop feedback so that the integrity of the system cannot be guaranteed by the rest faultless sensors. Thus, to design a fault-tolerant observer, when a close-loop control system has unstable outputs because of some faulty sensors, the observer can still obtain some state variables using the outputs of the rest faultless sensors and then guarantee the stability of the system and implement diagnosis. It has significant theoretical research value for enhancing the faulty tolerance of satellite attitude control systems and saving the resources of sensors.

At present, in order to accomplish the fault tolerant observation for sensor faults, there are two approaches: (i) the method based on the fault estimation compensation [3−5]; (ii) the method on the basis of linear transformation [6,7]. The first method is based on the fault residual error estimation and compensation, so it can only serve specific faults, and the estimation must be precise and real-time (the real-time estimation is on condition that the description of the fault and the model in the observer are exact). For the nonlinear sensor fault, the process of modeling is originally very complicated, and it demands the accurate quantitative estimation of faults; consequently, this method is not suitable for the fault tolerant observation of the uncertain model and the complicated sensor fault. The main idea of the second method firstly is to change the system into an equivalent triangle or observable subsystem through the linear transformation, and then design the observers according to the subsystems of the new system, which makes the output of observers decouple from faulty sensors; fi-

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nally achieve the fault observation by tracking the some system states. If we transform the original system and then design an observable subsystem observer, such as a dedicated observer [6], although it can satisfy the request of detecting the faulty residual error, it may not meet the need of the closed-loop feedback stability of the whole original system. Namely, it cannot give the feedback restrict condition of controllability and stability. Consequently, it just realizes fault detection and diagnosis (FDD), but ignoring the closed-loop stability. If we transform the original system into an equivalent triangle and then design, such as the method fault tolerant dimension reduction observer mentioned in [7], its observation results are irrelevant with the type of sensors' faults, but the precondition of system decomposition is that we must know that the output of which sensor is faulty, and design the transforming matrix according to the faulty sensor; in addition, it is for the open-loop system, not the feedback control.

According to the features of SACS, especially the closed-loop and real-time feedback of systems, we firstly design a fault observer to detect and isolate faulty sensors. Then the control system state equitation is transformed based on the correspondent transformation matrix, and then the triangular canonical form decomposition is done. The next step is to design a low-dimensional *KX* observer for different subsystems so as to obtain the original system *KX* observer with the fault-tolerance. By designing several parallel *KX* observers and the process of control switch, the SACS is able to guarantee the fault-tolerant observation of the remained system when some sensors have the unreliable output. Simulations illustrate the validity of this algorithm.

## 2. Problem description

Considering the linear time-invariant system, it can be described by the following state space model

$$\begin{cases} \dot{x}(t) = Ax(t) + Bu(t) \\ y(t) = Cx(t) \end{cases} \tag{1}$$

where $x \in \mathbb{R}^n$ is the input vector, $u \in \mathbb{R}^m$ is the control vector, and $y \in \mathbb{R}^l$ is the output vector. Without loss of generality, the parameter matrices $A$, $B$ and $C$ are obtained from the linearization at operation point for some practical systems. If we observe the system (1) with normal Luenberger dimension reduction observer, then through linear nonsingular transformation $x = pz$, we can transform the system into the following pattern [8]

$$\begin{cases} \begin{bmatrix} \dot{z}_1 \\ \dot{z}_2 \end{bmatrix} = \begin{bmatrix} \bar{A}_{11} & \bar{A}_{12} \\ \bar{A}_{21} & \bar{A}_{22} \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} + \begin{bmatrix} \bar{B}_1 \\ \bar{B}_2 \end{bmatrix} u \\ y = [0 \vdots \mathrm{I}] \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} \end{cases} \tag{2}$$

where $z_1$ is the $(n-l)$ dimension vector, and $z_2$ is the $l$ dimension vector. According to (2), we can get an $(n-l)$ dimensional state observer as follows:

$$\begin{cases} \dot{\hat{z}}_1 = \bar{A}_{11}\hat{z}_1 + \bar{A}_{12}y + \bar{B}_1 u \\ y = \bar{A}_{21}\hat{z}_1 \end{cases} . \tag{3}$$

Then, the state vector error is

$$\dot{e} = \dot{\hat{z}}_1 - \dot{z}_1 = \bar{A}_{11}(\hat{z}_1 - z_1) + \bar{A}_{12}(y - z_2). \tag{4}$$

Analyzing the observation process above, we find that traditional observers have the following problems: (i) if $\bar{A}_{12} \neq 0$, when the output is valid, $y = z_2$, and the state vector error reaches zero; when output is invalid, $y - z_2$ is not zero, state vector error does not reach zero, and the normal state observer of the system will be affected by the faulty output. So the Luenberger state observer is not fault-tolerant; (ii) if we choose the approachable $P$ transform matrix to make $\bar{A}_{12} = 0$, that is, a triangular canonical form, and the state vector error can reach zero exponentially. However, the triangular canonical form requires normal state as $z_1$, the state observation affected by the fault sensor. So isolating the faulty sensors is the precondition of transformation. This kind of transform is valid only for the specific fault output sensors; (iii) when the unreliable output of the system is isolated, the feedback stability cannot be guaranteed, which is hazardous for some important systems, especially the SACS. The proposed fault-tolerant observer method can solve the problems above.

## 3. Fault-tolerant observer methods

The two goals of the fault-tolerant observation are: (i) to decompose the original system into subsystems which are output-decoupling with each other; (ii) the feedback control, to the highest extent, based on the observation state of the reliable subsystem. To realize the above-mentioned goals, the fault residual error observers are employed to isolate and locate the fault of sensors. Then according to the isolation results, the original system is transformed into triangular canonical form by a correspondent $P$ matrix. After that the triangular canonical form of the original system is decomposed into subsystems, and then the *KX* subobservers are designed for each subsystem. The final stage is to design a *KX* observer for the whole system. In order to meet the need of stable closed-loop feedback control in both normal and faulty conditions, this paper utilizes a *KX* observer to directly observe the state feedback function *KX*, not the state $X$.

### 3.1 Fault observer

The function of a fault observer is to detect and isolate the specific sensor fault; therefore, this paper introduces a classical sensor fault residual error observer [9].

To design $l$ state observers, different signals measured by sensors are used as input signals for different observers. As shown in Fig. 1, the procedure is as follows:

(i) Dividing the output of the $l$ dimension sensor: $y = [y_1, y_2, \dots, y_l]^{\mathrm{T}}$ (T is on behalf of transpose, the same in the following article), in which $y_i$ is the output of the $i$th sensor $(i = 1, 2, \dots, l)$.

(ii) Building observers using $y_i$ and $u$; the $i$th observer can only be driven by $y_i$ and $u$.

(iii) Getting $l$ state estimation values $\hat{x}_1, \hat{x}_2, \dots, \hat{x}_l$ based on the observers above, $\hat{x}_i = [\hat{x}_{i1}, \hat{x}_{i1}, \dots, \hat{x}_{in}]^{\mathrm{T}}$ can be obtained from the $i$th observer $(i = 1, 2, \dots, l)$.

(iv) When the system runs normally, $\hat{x}_i$ $(i = 1, 2, \dots, l)$ should converge to the state $x$; when the fault happens on the $i$th sensor, the state value observed by the $i$th observer $\hat{x}_i$ would deviate the true state value, while the other $l - 1$ sensors run normally, leading to the following diagnostic conclusions.

Define a residual error

$$r_i = y_i - C_i\hat{x}_i, \quad i = 1, 2, \dots, l \tag{5}$$

where $C_i$ is the $i$th row of the output matrix. If $r_i < \varepsilon$, no fault occurs; if $r_i > \varepsilon$, the $i$th sensor goes wrong. Thereupon, the online fault detection and isolation can be performed.

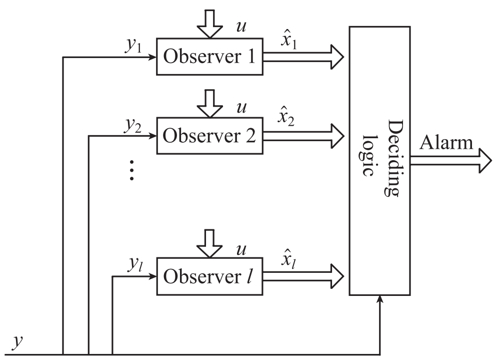


**Fig. 1 A fault observer**

In order to analyze the availability of the fault observer theoretically, Theorem 1 is given as below.

**Theorem 1** For the subsystem of object system (1),

$$\begin{cases} \dot{x}(t) = Ax(t) + Bu(t) \\ y_i(t) = C_i x(t) \end{cases}. \tag{6}$$

If the observing parameters $[A \quad C_i]$ satisfy the observable conditions, which is denoted as

$$\operatorname{rank}\,[C_i \quad C_iA \quad \dots \quad C_iA^{n-1}]^{\mathrm{T}} = n, \quad i = 1, \dots, l. \tag{7}$$

Then for its observer in such format,

$$\dot{\tilde{x}}(t) = A\tilde{x}(t) + Bu(t) + L_i(y_i(t) - C_i\tilde{x}(t)). \tag{8}$$

The parameter $L_i$ is available to define the poles of the parameter matrix $[A - L_iC_i]$ arbitrarily. Assuming each pole has a negative real part, the estimate state $\tilde{x}$ of observer (8) will be uniformly asymptotic to the system state $x$ when no faults occur. On the contrary, the estimate state $\tilde{x}$ will escape from the system state $x$ when a fault occurs.

**Proof** Let $e_x = x - \tilde{x}$ and compute the derivatives of its right and left side, and we get $\dot{e}_x = \dot{x} - \dot{\tilde{x}}$. Based on (6) and (8), the following is obtained

$$\begin{cases} \dot{e}_x(t) = (A - L_iC_i)e_x(t) \\ e_x(0) = x(0) - \tilde{x}(0) \end{cases}. \tag{9}$$

We get the solution of (9):

$$e_x(t) = e_x(0) \cdot \exp[(A - L_iC_i)t]. \tag{10}$$

Based on the duality principle between observability and controllability, the following is obtained.

$\{A,\, C_i\}$ is observable $\Rightarrow$ $\{A^{\mathrm{T}}, C_i^{\mathrm{T}}\}$ is controllable $\Rightarrow$ $K$ is available to definite the expected eigenvalue of$(A^{\mathrm{T}} + C_i^{\mathrm{T}}K)$ arbitrarily $\Rightarrow$ so the same is $(A + K^{\mathrm{T}}C_i)$.

Let $L_i = -K^{\mathrm{T}}$, then the eigenvalues of $(A - L_iC_i)$ can be set as the expected value. In other words, we can define the poles of $(A - L_iC_i)$ arbitrarily.

Assuming that the eigenvalues of $(A - L_iC_i)$ are set as negative for its real part, the following is obtained based on (10)

$$\lim_{t\to+\infty} e_x(t) = \lim_{t\to+\infty} [x(t) - \tilde{x}(t)] = 0. \tag{11}$$

Equation (11) means that the estimate state $\tilde{x}$ of observer (8) is uniformly asymptotic to the system state.

Considering the case with a sensor fault, system (1) can be changed as

$$\begin{cases} \dot{x}(t) = Ax(t) + Bu(t) \\ y_i(t) = C_i x(t) + f(t) \end{cases} \tag{12}$$

where $f(t)$ denotes the sensor fault function, and its value is zero during fault-free period while non-zero during fault period.

Substituting $\dot{e}_x = \dot{x} - \dot{\tilde{x}}$ with (8) and (12), the following is obtained

$$\begin{cases} \dot{e}_x(t) = (A - L_iC_i)e_x(t) + L_if(t) \\ e_x(0) = x(0) - \tilde{x}(0) \end{cases}. \tag{13}$$

As can be seen from (13), when no faults occur, $f(t) = 0$ and (13) is the same as (9), so $e_x(t)$ will be convergent to zero. When a fault occurs, the conditions of (10) do not hold, so $e_x(t)$ will escape from zero. □

Theorem 1 describes how to detect a sensor fault by using an observer with the corresponding subsystem. So if multiple observers are adopted to generate the residuals $r_i = y_i - C_i\hat{x}_i$ $(i = 1, 2, \dots, l)$, the fault sensor can be isolated.

In addition, if subsystem (6) does not satisfy the observable conditions, a replaceable solution is to decompose the unobservable system into an observable subsystem, and then an observer is designed for the subsystem using the same principle so as to overcome the limit of observable conditions. The details on observable decomposition are available in [10].

### 3.2 Triangular canonical form decomposition

Assuming that system (1) is observable, then its observation matrix $V = [C^{\mathrm{T}} A^{\mathrm{T}} C^{\mathrm{T}} \cdots (A^{n-1})^{\mathrm{T}} C^{\mathrm{T}}]^{\mathrm{T}}$ satisfies rank $(V) = n$. Let $C^{\mathrm{T}} = \left[c_1^{\mathrm{T}} c_2^{\mathrm{T}} \dots c_l^{\mathrm{T}}\right]$, we have

**Definition 1** If $v_1 = \mathrm{rank}\left[c_1^{\mathrm{T}} A^{\mathrm{T}} c_1^{\mathrm{T}} \dots (A^{n-1})^{\mathrm{T}} c_1^{\mathrm{T}}\right]$,

$$v_i = \mathrm{rank}[c_1^{\mathrm{T}} A^{\mathrm{T}} c_1^{\mathrm{T}} \dots (A^{v_1-1})^{\mathrm{T}} c_1^{\mathrm{T}} \dots c_{i-1}^{\mathrm{T}} A^{\mathrm{T}} C_{i-1}^{\mathrm{T}} \dots (A^{v_{i-1}-1})^{\mathrm{T}} c_{i-1}^{\mathrm{T}} c_i^{\mathrm{T}} A^{\mathrm{T}} c_i^{\mathrm{T}} \dots (A^{n-1})^{\mathrm{T}} c_i^{\mathrm{T}}] - \sum_{j=1}^{i-1} v_j,$$

$$i = 2, 3, \dots, l.$$

Then $\{v_i, i = 1, 2, \dots, l\}$ is called a triangular canonical form exponential set of system (1). Obviously, we have $\sum_{i=1}^{l} v_i = n$. Based on the triangular canonical form exponential set, we have Lemma 1.

**Lemma 1**[11] There exists a linear coordinate transformation $\bar{x} = Px$, which can transform system (1) as the triangular canonical form as follows:

$$\begin{cases} \dot{\tilde{x}}(t) = \widetilde{A}\tilde{x}(t) + \widetilde{B}u(t) \\ y(t) = \widetilde{C}\tilde{x}(t) \end{cases}. \tag{14}$$

The method of getting $P$ is as follows:

$$P^{-1} = [b_1\, Ab_1 \,\dots\, A^{v_1-1}b_1\, b_2\, Ab_2 \,\dots\, A^{v_2-1}b_2 \,\dots\, b_l\, Ab_l \,\dots\, A^{v_l-1}b_l]$$

where $b_i (i = 1, 2, \dots, l)$ is the solution of the following equation

$$b_i^{\mathrm{T}}\left[c_1^{\mathrm{T}} A^{\mathrm{T}} c_1 \dots (A^{v_1-1})^{\mathrm{T}} c_l^{\mathrm{T}} A^{\mathrm{T}} c_l^{\mathrm{T}} \dots (A^{v_l-1})^{\mathrm{T}} c_l^{\mathrm{T}}\right] = [0 \,\dots\, 0\; 1\; 0 \,\dots\, 0].$$

The value of $\sum_{j=1}^{i} v_j$ is the sequence number of the formula above.

Assume rank $C = l$ and $1 \leqslant \widetilde{l} < l$. Let $\tilde{x}^{\mathrm{T}} = [\tilde{x}_1^{\mathrm{T}} \quad \tilde{x}_2^{\mathrm{T}}]$, $\tilde{y}^{\mathrm{T}} = [\tilde{y}_1^{\mathrm{T}} \quad \tilde{y}_2^{\mathrm{T}}]$, $\tilde{x}_1 \in \mathbb{R}_{\tilde{v}_1}, \tilde{x}_2 \in \mathbb{R}_{\tilde{v}_2}, \tilde{y}_1 \in \mathbb{R}_l, \tilde{y}_2 \in \mathbb{R}_{l-\tilde{l}}$, $\tilde{v}_k = \sum_{i=1}^{l} v_i$, $\tilde{v}_2 = \sum_{i=\tilde{l}+1}^{l} v_i$, and $v_i = n - \tilde{v}_1$, then system (14) can be expressed as

$$\begin{aligned} &\begin{bmatrix} \dot{\tilde{x}}_1 \\ \dot{\tilde{x}}_2 \end{bmatrix} = \begin{pmatrix} \tilde{A}_1 & 0 \\ \tilde{A}_2 & \tilde{A}_3 \end{pmatrix} \begin{bmatrix} \tilde{x}_1 \\ \tilde{x}_2 \end{bmatrix} + \begin{bmatrix} \tilde{B}_1 \\ \tilde{B}_2 \end{bmatrix} u \\ &\begin{bmatrix} \tilde{y}_1 \\ \tilde{y}_2 \end{bmatrix} = \begin{pmatrix} \tilde{C}_1 & 0 \\ \tilde{C}_2 & \tilde{C}_3 \end{pmatrix} \begin{bmatrix} \tilde{x}_1 \\ \tilde{x}_2 \end{bmatrix}, \begin{bmatrix} \tilde{x}_1(0) \\ \tilde{x}_2(0) \end{bmatrix} = \begin{bmatrix} \tilde{x}_{10} \\ \tilde{x}_{20} \end{bmatrix} \end{aligned}. \tag{15}$$

Let $\tilde{B}_2' = [\tilde{A}_2 \quad \tilde{B}_2], u' = [\tilde{x}_1^{\mathrm{T}} \quad u^{\mathrm{T}}]^{\mathrm{T}}$, and $\tilde{y}_2' = \tilde{y}_2 - \tilde{C}_2\tilde{x}_1$, then system (15) can be decomposed into the two subsystems

$$\begin{cases} \dot{\tilde{x}}_1 = \tilde{A}_1\tilde{x}_1 + \tilde{B}_1 u, \quad \tilde{x}_1(0) = \tilde{x}_{10} \\ \tilde{y}_1 = \tilde{C}_1\tilde{x}_1 \end{cases} \tag{16}$$

$$\begin{cases} \dot{\tilde{x}}_2 = \tilde{A}_3\tilde{x}_2 + \tilde{B}_2' u', \quad \tilde{x}_2(0) = \tilde{x}_{20} \\ \tilde{y}_2' = \tilde{C}_3\tilde{x}_2 \end{cases}. \tag{17}$$

### 3.3 *KX* function observer

In the state feedback, the control law can be generally expressed as $u = -Kx$. In order to reduce the dimension of observers, *KX* observers can be used to reconstruct to directly observe the function of state variables $Kx$. As for the designing method and proof, refer to [12,13], this paper just gives the result: establish the following observer and set $Kx$ as the observer objective

$$\begin{cases} \dot{z} = Fz + Gy + Hu \\ \omega = Mz + Ny \end{cases}. \tag{18}$$

This observer satisfies

$$\begin{cases} \lim\limits_{t\to\infty} \omega(t) = \lim\limits_{t\to\infty} Kx(t) \\ \lim\limits_{t\to\infty} z(t) = \lim\limits_{t\to\infty} Tx(t) \end{cases}$$

the necessary and sufficient conditions are

(i) $T'A - FT' = GC$ ($T'$ is a real constant matrix);

(ii) $H = T'B$;

(iii) All characteristic values of $F$ have the negative real part;

(iv) $MT + NC = K$.

### 3.4 Equivalence of subsystem and original system

After obtaining the system triangular canonical form through the linear coordinate transformation $\bar{x} = Px$ on the original system, according to the feedback parameters $K$, choose suitable $T', F, G, H, M, N$ which satisfy the condition (i)$-$condition (iv) above, and a system observer taking $Kx$ as the objective can be designed. In order to make the system have the ability of fault tolerance, solving the parameters above should be restricted. The main idea of solving is decomposing the system into a triangular canonical form according to the output faulty. Assuming that the output $\tilde{y}_2^{\mathrm{T}}$ of $\tilde{y}^{\mathrm{T}} = [\tilde{y}_1^{\mathrm{T}} \quad \tilde{y}_2^{\mathrm{T}}]$ is unreliable. Then taking system (16) as the normal system, and system (17) as the faulty subsystem, design the observers based on the two systems respectively; on the basis of $T_i', F_i, G_i, H_i, M_i, N_i$ $(i = 1, 2)$ in each subsystem, the $T', F, G, H, M, N$ of system can be calculated. The parameters of subsystem and system satisfy the following relationship.

If the parameters of the original system $Kx$ can be expressed as

$$T' = \begin{bmatrix} T_1' & 0 \\ T_3' & T_4' \end{bmatrix}, \quad F = \begin{bmatrix} F_1 & 0 \\ 0 & F_2 \end{bmatrix}, \quad G = \begin{bmatrix} G_1 & 0 \\ G_3 & G_4 \end{bmatrix}$$

$$H = \begin{bmatrix} H_1 \\ H_2 \end{bmatrix}, \quad M = [M_1 \quad M_2], \quad N = [N_1 \quad N_2]$$

and the parameters of $Kx$ observer of subsystem (16) can be expressed as $\tilde{T}_1', \tilde{F}_1, \tilde{G}_1, \tilde{H}_1, \tilde{M}_1, \tilde{N}_1$, while the parameters of $Kx$ observer of subsystem (17) can be expressed as $\tilde{T}_2', \tilde{F}_2, \tilde{G}_2, \tilde{H}_2, \tilde{M}_2, \tilde{N}_2$, then the following equations should be satisfied.

(i) $T_1' = \tilde{T}_1', T_4' = \tilde{T}_2'$;
(ii) $F_1 = \tilde{F}_1, F_2 = \tilde{F}_2$;
(iii) $G_1 = \tilde{G}_1, G_4 = \tilde{G}_2$;
(iv) $H_1 = T_1'\tilde{B}_1, H_2 = T_3'\tilde{B}_1 + T_4'\tilde{B}_2$;
(v) $M_1 = \tilde{M}_1, M_2 = \tilde{M}_2$;
(vi) $N_1 = \tilde{N}_1, N_2 = \tilde{N}_2$;
(vii) $G_3$, $T_3'$ are arbitrarily chosen but should satisfy $T_3'\tilde{A}_1 + T_4'\tilde{A}_2 - F_2T_3' = G_3\tilde{C}_1 + G_4\tilde{C}_2$.

## 4. Fault-tolerant observation on SACS

The SACS is a highly complicated closed-loop feedback system, where the outputs of sensors interact, and the feedback signals of fault sensors may bring the closed-loop system down; therefore, it is necessary to design a fault tolerant observer to restrain the effect of unreliable outputs on other reliable outputs and to guarantee the stability of the closed-loop feedback.

### 4.1 Mathematical description of SACS

Considering the jet control of the earth-oriented three-axis stabilized satellite, we take the satellite as a rigid body. Because the pitching channel is decoupling, it can be designed alone [14]. Therefore, here we just consider the state space form of the rolling and yawing orbit,

$$\begin{cases} \begin{bmatrix} \ddot{\phi} \\ \ddot{\psi} \end{bmatrix} = \begin{pmatrix} 0 & \dfrac{(I_y - I_z)\omega_0}{I_x} \\ \dfrac{(I_x - I_y)\omega_0}{I_z} & 0 \end{pmatrix} \begin{bmatrix} \dot{\phi} \\ \dot{\psi} \end{bmatrix} + \\ \qquad \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{bmatrix} L_x \\ L_z \end{bmatrix} \\ \begin{bmatrix} \dot{\phi} \\ \dot{\psi} \end{bmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{bmatrix} \dot{\phi} \\ \dot{\psi} \end{bmatrix} \end{cases} \tag{19}$$

where $L_x, L_y$, and $L_z$ are the three components of the external torque in the satellite ontology coordinate system; $I_x$, $I_y$, and $I_z$ are three main inertias of the satellite; and $\omega_0$ is the angular velocity of the satellite.

To facilitate the following exposition, here let

$$x = \begin{bmatrix} \dot{\phi} \\ \dot{\psi} \end{bmatrix}, \quad y = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} \dot{\varphi} \\ \dot{\psi} \end{bmatrix}$$
$$u = \begin{bmatrix} L_x \\ L_z \end{bmatrix}, \quad C = I_{2\times 2}$$

$$A = \begin{pmatrix} 0 & \dfrac{(I_y - I_z)\omega_0}{I_x} \\ \dfrac{(I_x - I_y)\omega_0}{I_z} & 0 \end{pmatrix}, \quad B = \begin{bmatrix} L_x \\ L_z \end{bmatrix}.$$

Then the attitude control system can be expressed as the state equation of system (1).

### 4.2 Fault tolerant observer and *KX* feedback control

The fault-tolerant and feedback control of rolling/ yawing-loop are shown in Fig. 2. The flow includes two parts, a offline stage for observer design (shown in dotted -line box) and a online observing stage (shown in solid-line box).

During the offline stage, an FD observer and *KX* observers are designed based on system state equations and the *K*-feedback control law. The FD observer is based on the steps in Section 3.1. *KX* observers in different fault modes are implement by four steps: (i) triangular canonical form transformation; (ii) subsystem decomposition; (iii) normal and fault *KX* subobserver design; (iv) whole *KX* observer design.

During the online stage, the FD observer and *KX* observer designed offline are simultaneously embedded to monitor and observe the system state online. The inputs of two observers are the same, which are system sensor output $y$ and actuator output $u$. The output of the *KX* observer is the function of state which is denoted as *KX*. The output of the FD observer is the residuals which are denoted as $r_i$ $(i = 1, 2)$. These residuals can be used to diagnose which a sensor is fault. For the control switching function, the output is either an output of the *KX* observer 1 or an output of the *KX* observer 2, which is selected depending on the FD result based on the residuals $r_i$ $(i = 1, 2)$. Furthermore, if the residuals indicate sensor 1 is fault, the output of the *KX* observer 1 is selected; else if the residuals indicate sensor 2 is fault, the output of the *KX* observer 2 is selected. The default output is the one of *KX* observer 1.

Note that the *KX* observer 1 and *KX* observer 2 must refer to different descriptions of the system state. According to the *KX* observation theory, a *KX* observer is valid only when the output $y_2$ is unreliable; therefore, for the *KX* observer 1, the state should be described as $x = \begin{bmatrix} \dot{\psi} \\ \dot{\phi} \end{bmatrix}$, and its corresponding state space equation should also be done with a row transformation, and then the original system is transformed into the equivalent triangular canonical form using $\bar{x} = Px$, in which matrix $P$ must be constructed to make correspondent $\bar{A}_{12} = 0$.

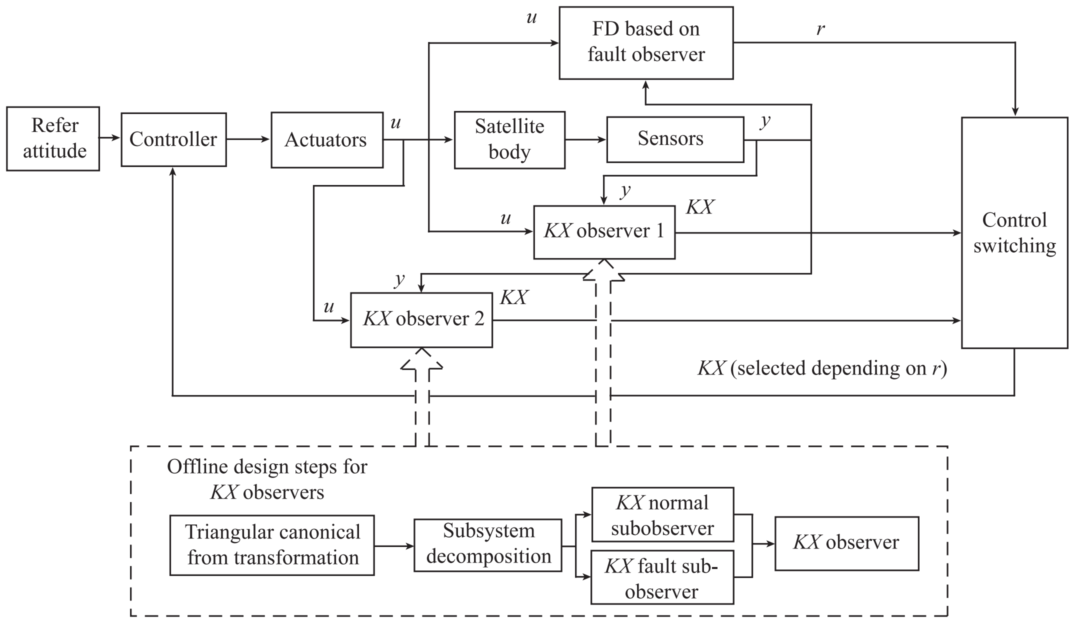


**Fig. 2 Flow chart of satellite attitude fault-tolerant observation and control**

Whichever *KX* observer it is, the observer is always able to track the partial normal state of the system when the sensor output is unreliable, and the corresponding K control law can be arbitrarily choosed once the system close-loop poles are in the left half-plane. So, it is easy to meet the closed-loop stability of the partial state feedback, and the constraint condition of K for controllability is detailed in [13]. Thus, it ensures the closed-loop stability control of the system whether in the normal state or in the condition of a failure in either sensor.

## 5. Experimental analysis

This paper specifies the parameters of the satellite attitude control system are as follows: the three principal inertia moments of the satellite to its mass center are $I_x = 80\ \mathrm{kgm}^2$, $I_y = 90\ \mathrm{kgm}^2$ and $I_z = 70\ \mathrm{kgm}^2$; satellite orbital angular velocity $\omega_0 = 0.001$ rad/s. Based on the analysis in Section 4.1, the state space expression of the jet control system is expressed as

$$\begin{bmatrix} \ddot{\phi} \\ \dot{\phi} \\ \ddot{\psi} \\ \dot{\psi} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0.001\,25 & 0 \\ 1 & 0 & 0 & 0 \\ -0.000\,143 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} \dot{\varphi} \\ \varphi \\ \dot{\psi} \\ \psi \end{bmatrix} + \begin{bmatrix} 0.012\,5 & 0 \\ 0 & 0 \\ 0 & 0.014\,3 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} L_x \\ L_z \end{bmatrix}$$

$$\begin{bmatrix} \dot{\varphi} \\ \varphi \\ \dot{\psi} \\ \psi \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \dot{\varphi} \\ \varphi \\ \dot{\psi} \\ \psi \end{bmatrix} \tag{20}$$

where a classical PD control mode is taken as the control law, that is

$$u = -Kx = -[Kp_1 \quad Kd_1 \quad Kp_2 \quad Kd_2] \otimes [\dot{\varphi} \quad \varphi \quad \dot{\psi} \quad \psi]^{\mathrm{T}}.$$

In order to show the superiority of the fault-tolerant observation scheme proposed in the paper compared with other schemes, comparative simulation experiments based on the jet control system mentioned above are carried out in two aspects: state observation fault-tolerant performance (whether the fault output affects the normal state); the overall fault-tolerant and closed-loop stability performance of sensor failures (whether it is tolerant for all sensor failures).

Three fault scenarios are considered to validate the proposed method.

**Fault scenario 1** A step failure occurs at 1 s for sensor $2(y_2)$.

**Fault scenario 2** Sensor $2(y_2)$ runs an intermittent failure between 4 s and 4.5 s.

**Fault scenario 3** Sensor $1(y_1)$ runs an intermittent failure between 3.5 s and 3.55 s.

### 5.1 State observation fault-tolerant performance comparison

In order to validate the effect of the presented observer method in the paper for system (12), simulations in fault and fault-free mode are performed and compared.

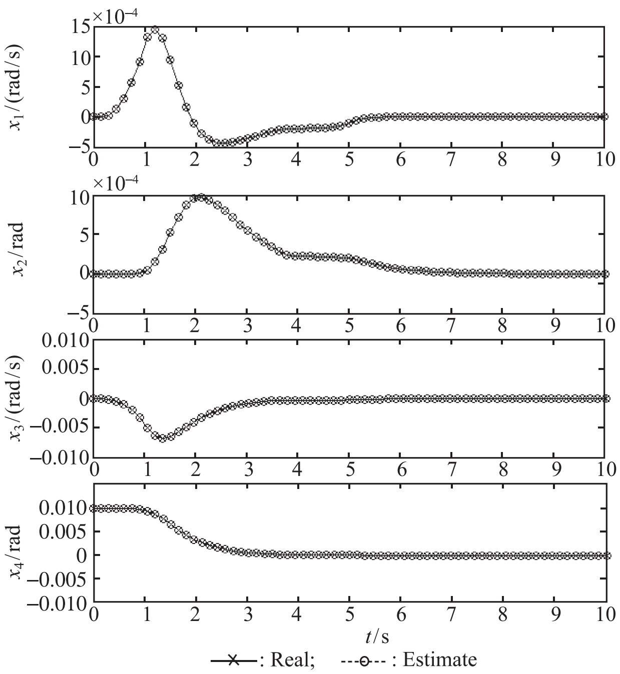


**Fig. 3 The estimate of improved *KX* fault-tolerant observer in the fault-free mode**

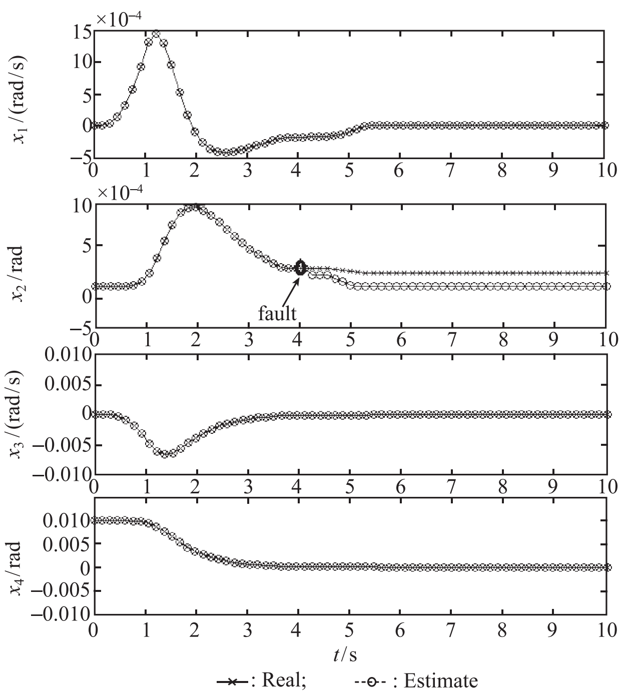


**Fig. 4 The estimate of improved *KX* fault-tolerant observer in fault scenario 1**

In our simulation, the observing state of sensor 1 includes $x_1$ and $x_2$. The observing state of sensor 2 includes $x_3$ and $x_4$ (the same in the remaining part). $x_1$ denotes yaw angle velocity; $x_2$ denotes a yaw angle; $x_3$ denotes rolling angle velocity; $x_4$ denotes a rolling angle; and $t$ denotes the simulation running time.

Assuming that a step failure occurs at 1 s for sensor 2 (named as fault scenario 1), we obtain state observation curves in fault-free and fault mode as shown in Fig. 3 and Fig. 4, respectively.

From Fig. 3, we can see that the improved *KX* fault-tolerant observer presented can track all the states of the object system. And Fig. 4 indicates that when a step failure occurs for sensor 2 at 4 s, the observer presented can still track $x_1$, $x_3$ and $x_4$ except $x_2$ because of the effects of the faulty sensor. Thus it can be seen that when partial outputs are not reliable the observer presented is still capable of tracking remaining normal states.

### 5.2 Whole fault-tolerant performance and closed loop stability comparison of sensor failures

The improved *KX* observer method in the paper is compared with the unimproved *KX* observer method (just for one sensor) in terms of the tracked states in the condition of different sensor failures. Assume that sensor 2 runs an intermittent failure between 4 s and 4.5 s, and sensor 1 runs an intermittent failure between 3.5 s and 3.55 s. The state observation curves of both observers in fault scenario 2 are shown in Fig. 5 and Fig. 6, and the state observation curves of both observers in fault scenario 3 are shown in Fig. 7 and Fig. 8.

It is seen from Fig. 5 and Fig. 6 that when no fault occurs, both *KX* observer and improved *KX* fault-tolerant observer can track all states; when sensor 2 is wrong between 4 s and 4.5 s, the *KX* observer cannot track all the system state any more but the improved *KX* observer can still track the real state variables $x_1$, $x_3$ and $x_4$ except $x_2$. Thus it can be seen that the improved *KX* observer is fault-tolerant for sensor 2.

It is seen from Fig. 7 and Fig. 8 that when no fault occurs, both *KX* observer and improved *KX* fault-tolerant observer can track all states; when sensor 1 is wrong between 3.5 s and 3.55 s, the *KX* observer cannot track all the system state any more but the improved *KX* observer can still track the real state variables $x_1$, $x_2$ and $x_3$ except $x_4$. Thus it can be seen that the improved *KX* observer is fault-tolerant for sensor 1.

Moreover, by comparing with the lines after the fault disappears in Fig. 6 and Fig. 8, we can see that although there exists fault in both sensor 1 and sensor 2, the improved *KX* observer can still track the partial state of the real system and keep the close-loop stable.

Based on the analysis above, we conclude that the improved *KX* fault-tolerant observer method is able to estimate remaining normal system states and ensure the stability of the closed-loop feedback control system in condition of failures in either sensor as well. Therefore, both its fault-tolerant observation performance and feedback system stability are superior to the other two observer methods.

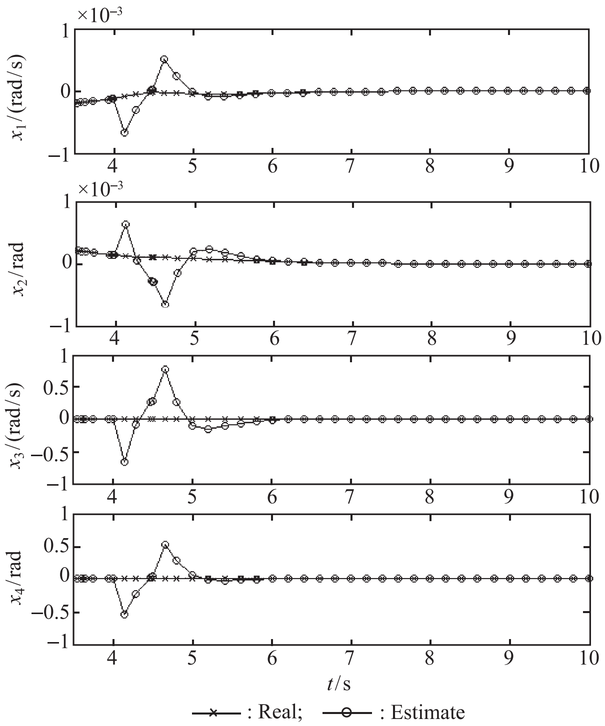


**Fig. 5 The estimate of *KX* observer in fault scenario 2 (fault-tolerant for sensor 1)**

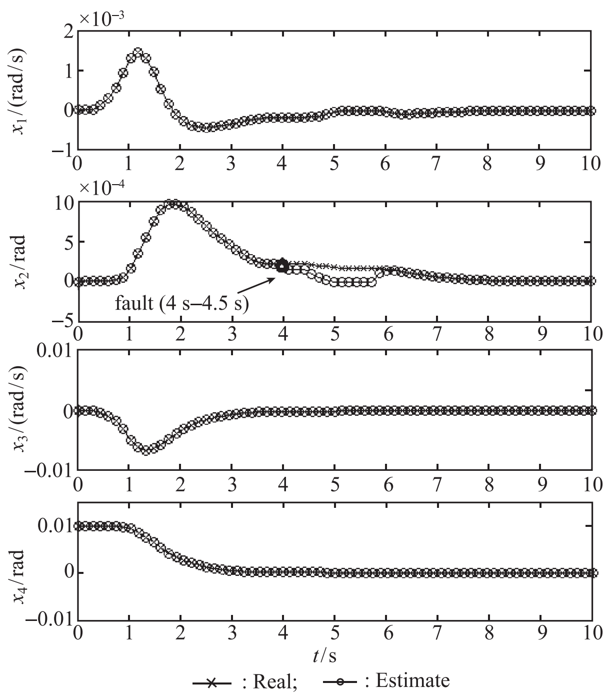


**Fig. 6 The estimate of improved *KX* fault-tolerant observer in fault scenario 2**

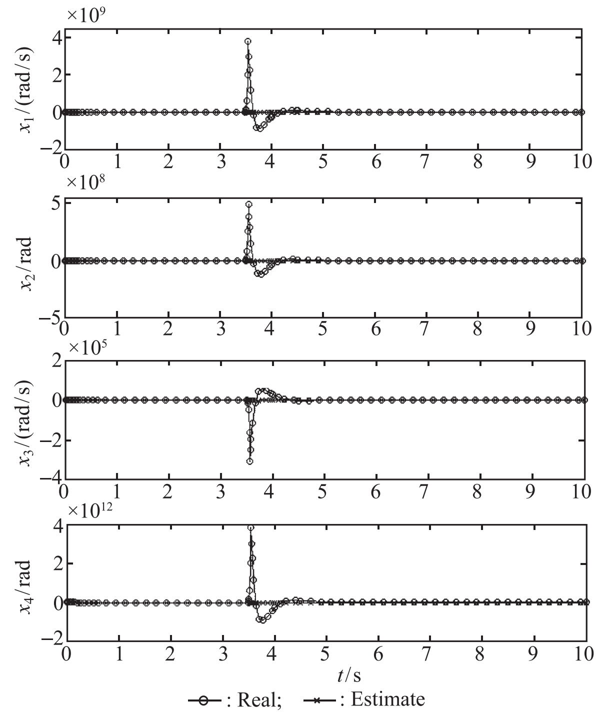


**Fig. 7 The estimate of *KX* observer in fault scenario 3 (fault-tolerant for sensor 2)**

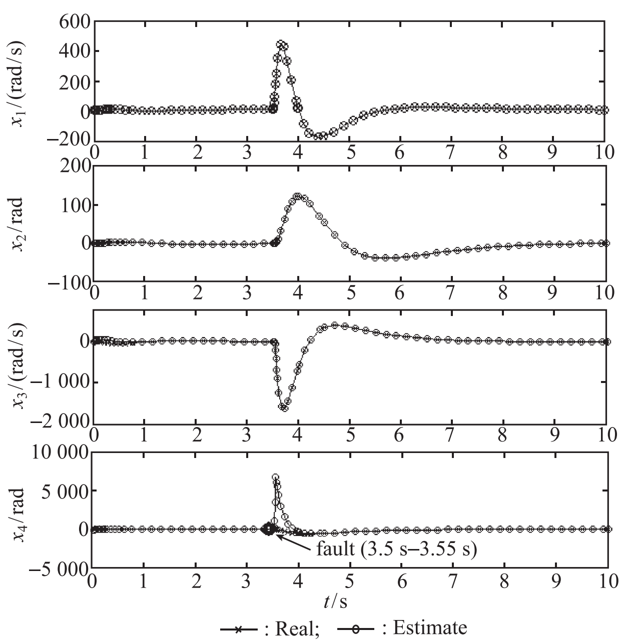


**Fig. 8 The estimate of improved *KX* fault-tolerant observer in fault scenario 3**

## 6. Conclusion

In this paper, a fault observer is introduced based on the unimproved *KX* observer method for the fault detection and isolation; parallel *KX* observers and a process of control switch are designed for different unreliable sensors; additionally, the combination of *KX* observer design and the control law of closed-loop stability feedback together enables the control system to ensure the fault-tolerant observation of remaining partial observations of the system even when part of sensor outputs are unreliable. The *KX*-based feedback mechanism ensures the integrity of closed-loop control of the fault system. The fault-tolerant observation design method, which is simulated and verified in a certain type of the satellite attitude control system, is superior to other observer methods in terms of relevant performance comparisons, and it can implement the fault-tolerant observation and stability control. Thus the expected purposes have been achieved.

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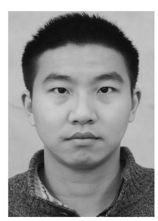

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